SOLUTION

PHYSICS

1.(c)

$\mathbf{u} = 3\hat{i} + 4\hat{j}, \mathbf{a} = 0.4\hat{i} + 0.3\hat{j}$

Speed $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$= 3\hat{i} + 4\hat{j} + (0.4\hat{i} + 0.3\hat{j})10$

$= 3\hat{i} + 4\hat{j} + 4\hat{i} + 3\hat{j} = 7\hat{i} + 7\hat{j}$

$v = \sqrt{7^2 + 7^2} = 7\sqrt{2}$ unit

2(b)

$$t = \frac{210}{25+5} = 7s$$

3(c)

$10^2 = 8^2 + v_r^2$

$\Rightarrow v_r = 6$ km/hr

4(a)

$$\sin\theta = \frac{2}{3} \quad \Rightarrow \quad \cos\theta = \frac{\sqrt{5}}{3}$$

$$t = \frac{d}{v\cos\theta} = \frac{0.500}{3\cos\theta} = \frac{\sqrt{5}}{10}\text{hr}$$

5(c)

$$t = \frac{d}{v_r} = \frac{0.500}{3\cos 30^\circ} = \frac{1}{3\sqrt{3}}\text{hr}$$

6(b)

$v_{bw} = 3\hat{i} + 4\hat{j} - (-3\hat{i} - 4\hat{j})$

$v_{bw} = 6\hat{i} + 8\hat{j}$

7 (d)

Total distance = 130 + 120 = 250 m

Relative velocity = 30 - (-20) = 50 m/s

Hence, t = 250/50 = 5s

8 (d)

Relative velocity of police man w.r.t. the time $10 - 9 = 1\text{ms}^{-1}$. Since the relative separation between them is 100 m, hence, the time taken will be = relative separation/relative velocity =100/1=100s

9 (b)

Boat covers distance of 16 km in a still water in hours

ie $v_B = \frac{16}{2} = 8\text{kmh}^{-1}$

Now, velocity of water

$v_W = 4\text{kmh}^{-1}$

Time taken for going upstream

$t_1 = \frac{8}{v_B - v_W} = \frac{8}{8-4} = 2\text{h}$

(As water current oppose the motion of boat)

Time taken for going downstream

$t_2 = \frac{8}{v_B + v_W} = \frac{8}{8+4} = \frac{8}{12}\text{h}$

(As water current helps the motion of boat)

$\therefore$ Total time $= t_1 + t_2$

$= \left(2 + \frac{8}{12}\right)$h=2 h 40 min

10. (c)

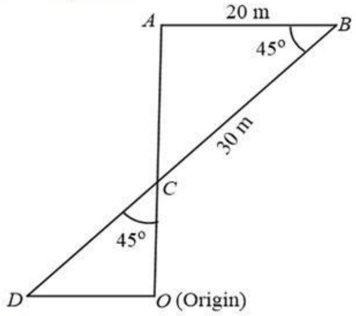
Time to cross the river in shortest time is

$$t = \frac{w}{\sqrt{v_B^2 - v_R^2}}$$

or $\frac{20}{60} = \frac{1}{\sqrt{25 - v_R^2}}$ or $v_R = 4$ km/hr

11. (c)

Taking the starting point as O, we have 30 m north OA, 20 m east AB, and finally $30\sqrt{2}$ m (S – W)BD.



From Δ CAB,

AC = 20 m, OC = 10 m

In Δ OCD,

OD = OC, OD = 10 m

Hence, final displacement from origin is 10 m.

12. **(a)**

$\tan(90^\circ - \theta) = \frac{20}{15}$

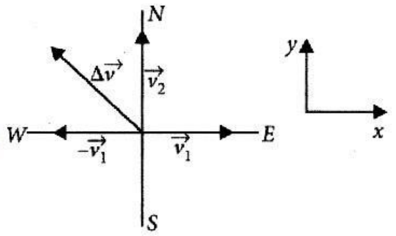
$\therefore \cot\theta = \frac{20}{15} = \frac{4}{3}$

$\Rightarrow \theta = 37^\circ$

$\therefore \theta = 37^\circ + 23^\circ$

$= 60^\circ$

13.

(d) : 

Velocity towards east direction, $\vec{v}_1 = 30\,\hat{i}$ m/s

Velocity towards north direction, $\vec{v}_2 = 40\,\hat{j}$ m/s

Change in velocity, $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = (40\,\hat{j} - 30\,\hat{i})$

$\therefore\ |\Delta\vec{v}| = |40\hat{j} - 30\hat{i}| = 50$ m/s

Average acceleration, $\vec{a}_{av} = \dfrac{\text{Change in velocity}}{\text{Time interval}}$

$\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \dfrac{\Delta\vec{v}}{\Delta t}$

$|\vec{a}_{av}| = \dfrac{|\Delta\vec{v}|}{\Delta t} = \dfrac{50 \text{ m/s}}{10 \text{ s}} = 5 \text{ m/s}^2$

14. **(b)**

$F^2 = F^2 + F^2 + 2F^2\cos\theta$

or $F^2 = 2F^2(1 + \cos\theta)$

or $1 + \cos\theta = \frac{1}{2}$

or $\cos\theta = -\frac{1}{2}$ or $\theta = 120^\circ$

$\therefore \cos 120^\circ = -\dfrac{1}{2}$

15. **(c)**

$\vec{A}\cdot\vec{B} = AB\cos\theta = 6$

and $|\vec{A}\times\vec{B}| = AB\sin\theta = 6\sqrt{3}$

$\therefore \dfrac{AB\sin\theta}{AB\cos\theta} = \dfrac{6\sqrt{3}}{6} = \sqrt{3}$

or $\tan\theta = \sqrt{3}$

and $\theta = 60^\circ$

16. **(c)**

$A_x = 50, \theta = 60^\circ$

Then $\tan\theta = A_y/A_x$ or $A_y = A_x\tan\theta$

Or $A_y = 50\tan 60^\circ = 50 \times \sqrt{3} = 87$ N

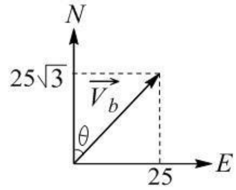
17. **(c)**

$\vec{v}_{r/g} = \vec{v}_r + (-\vec{v}_g)\vec{v}_r - \vec{v}_g = -4\hat{j} - 3\hat{i}$

$v_{\frac{r}{g}} = \sqrt{v_r^2 + v_g^2} = \sqrt{16 + 9}\text{km h}^{-1} = 5 \text{ km h}^{-1}$

18. **(a)**

$\vec{v}_c = 25\hat{i}, \vec{v}_{b/c} = 25\sqrt{3}\hat{j}$



$\vec{V}_{b/c} = \vec{V}_b - \vec{V}_c \Rightarrow \vec{V}_b = \vec{V}_{b/c} + \vec{V}_c \Rightarrow \vec{v}_b$

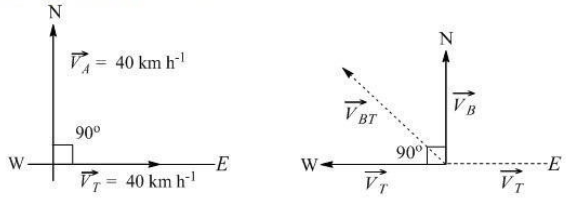
$= 25\hat{i} + 25\sqrt{3}\hat{j}$

$|\vec{v}_b| = \sqrt{25^2 + (25\sqrt{3})^2} = 50 \text{ km h}^{-1}$

$\tan\theta = \dfrac{25}{25\sqrt{3}} = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$

19. **(c)**

To find the relative velocity of bird w.r.t. train, superimpose velocity $-\vec{V}_T$ on both the object. Now as a result of it, the train is at rest, while bird possesses two velocities, $\vec{V}_B$ towards north and $\vec{V}_T$ along west



$|\vec{V}_{BT}| = \sqrt{|\vec{V}_B|^2 + |-\vec{V}_T|^2}$ [By formula, $\theta = 90^\circ$]

$= \sqrt{40^2 + 40^2} = 40\sqrt{2}$ km h^{-1} north-west

20. **(a)**

$(2\hat{i} - 3\hat{j} + \hat{k}).(3\hat{i} + 3\hat{j}) = 6(\hat{i}.\hat{i}) - 6(\hat{j}.\hat{j}) = 0$

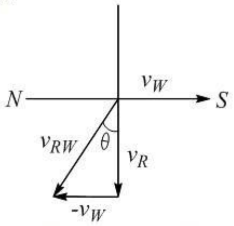
21. **(a)**

Here, $v_R = 25 \text{ ms}^{-1}, v_w = 10 \text{ ms}^{-1}$

Velocity of rain w.r.t. women: $v_{R/W} = v_R - v_w$

Let $v_{R/W}$ make an angle θ with vertical, then

$\tan\theta = \frac{v_w}{v_R} = \frac{10}{2.5} = 0.4$



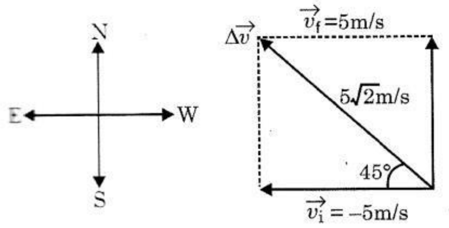
She should hold her umbrella at an angle of $\theta = \tan^{-1}(0.4)$ with the vertical towards south

22. **(b) :** Since the angular momentum has both magnitude and direction, it is a vector quantity.

23. **(c)**

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$\Delta\vec{v} = 5\sqrt{2}$ m/s in north - west direction.



$$\vec{a}_{av} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}} m/s^2 \text{ (in north - west direction)}$$

Correct option is (c).

24. **(c)**

$x + y = 16$, Also $y^2 = 8^2 + x^2$

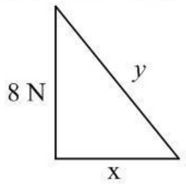
or $y^2 = 64 + (16 - y)^2$

$[\because \ x = 16 - y]$

or $y^2 = 64 + 256 + y^2 - 32y$

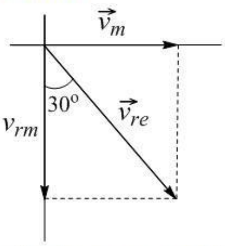
or $32y = 320$ or $y = 10N$

$\therefore \ x + 10 = 16$ or $x = 6N$



25. **(a)**

$\vec{v}_m$ = Velocity of man



$\vec{v}_{re}$ = Velocity of rain w.r.t. earth

$\vec{v}_{rm}$ = Velocity of rain w.r.t. man

Velocity of man $|\vec{v}_m| = 10 \text{ ms}^{-1}$

Using $\sin 30° = \frac{v_m}{v_{re}}$

$$v_{re} = \frac{v_m}{\sin 30°} = \frac{10}{1/2} = 20 \text{ ms}^{-1}, \cos 30° = \frac{v_{rm}}{v_{re}}$$

$$v_{rm} = v_{re}\cos 30 = \frac{20 \times \sqrt{3}}{2} = 10\sqrt{3} \text{ ms}^{-1}$$

26. **(d) :** Positive vector of the particle

$\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$

velocity vector

$$\vec{v} = \frac{d\vec{r}}{dt} = (-a\omega\sin\omega t)\hat{i} + (a\omega\cos\omega t)\hat{j}$$

$$= \omega[(-a\sin\omega t)\hat{i} + (a\cos\omega t)\hat{j}]$$

$$\vec{v}.\vec{r} = \omega[(-a\sin\omega t)\hat{i} + (a\cos\omega t)\hat{j})].$$

$$[(a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j})]$$

$$= \omega[-a^2\sin\omega t\cos\omega t + a^2\cos\omega t\sin\omega t] = 0$$

Therefore velocity vector is perpendicular to the displacement vector.

27. **(a) :** $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$$

$$= \frac{(3\hat{i} + 4\hat{j} + 5\hat{k}).(3\hat{i} + 4\hat{j} - 5\hat{k})}{[\sqrt{(3)^2 + (4)^2 + (5)^2}]\times[\sqrt{(3)^2 + (4)^2 + (5)^2}]}$$

$$= \frac{9 + 16 - 25}{50} = 0 \text{ or } \theta = 90^o.$$

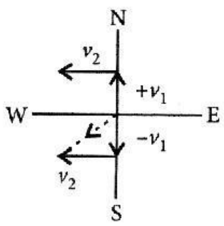
28. **(a) :** Let θ be angle between $\vec{A}$ and $\vec{B}$

Given : $A = |\vec{A}| = 3$ units

$B = |\vec{B}| = 4$units

$C = |\vec{C}| = 5$units

$\vec{A} + \vec{B} = \vec{C}$



$\therefore (\vec{A} + \vec{B}).(\vec{A} + \vec{B}) = \vec{C}.\vec{C}$

$\vec{A}.\vec{A} + \vec{A}.\vec{B} + \vec{B}.\vec{A} + \vec{B}.\vec{B} = \vec{C}.\vec{C}$

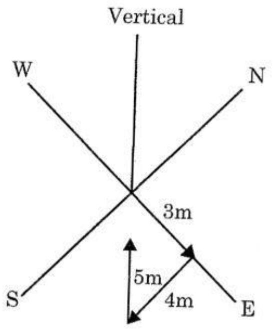
$A^2 + 2AB\cos\theta + B^2 = C^2$

$9 + 2AB\cos\theta + 16 = 25$ or $2AB\cos\theta = 0$

or $\cos\theta = 0 \therefore \theta = 90°$

29. **(b)**

Displacement of the girl is shown below



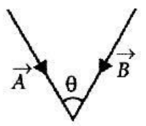
So magnitude of her displacement is

$= \sqrt{3^2 + 4^2 + 5^2} = 5\sqrt{2}\text{m}$

30. **(d) :** Let $\vec{A} \times \vec{B} = \vec{C}$

The cross product of $\vec{B}$ and $\vec{A}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ i.e. perpendicular to $\vec{A}$.

If a dot product of this cross product and $\vec{A}$ is taken, as the cross product is perpendicular to $\vec{A}, \vec{C} \times \vec{A} = 0$.



Therefore product of $(\vec{B} \times \vec{A}).\vec{A} = 0$.

31. **(b) :** $\vec{a} = 2\hat{i} + 3\hat{j} + 8\hat{k}, \vec{b} = 4\hat{j} - 4\hat{i} + \alpha\hat{k}$

$\vec{a}.\vec{b} = 0$ if $\vec{a} \perp \vec{b}$

$(2\hat{i} + 3\hat{j} + 8\hat{k}).(-4\hat{i} + 4\hat{j} + \alpha\hat{k}) = 0$

or, $-8 + 12 + 8\alpha = 0 \Rightarrow 4 + 8\alpha = 0$

$\Rightarrow \alpha = -1/2$.

32. **(a) :** $v_{\text{Resultant}} = \dfrac{1 \text{ km}}{1/4 \text{ hr}} = 4 \text{ km/hr}$

$\therefore v_{\text{River}} = \sqrt{5^2 - 4^2} = 3 \text{ km/hr}$

33. **(d)**

$|\vec{a} \times \vec{b}| = ab \sin\theta$

$\sin\theta$ cannot be greater than 1.

$\therefore |\vec{a} \times \vec{b}|$ cannot be greater than ab.

34. **(b)**

$\vec{r} = at^2\hat{i} + bt\hat{j}$

$x = at^2$ ...(i)

and $y = bt$ ... (ii)

From Eq. (ii) put value of t in Eq. (i)

$x = \dfrac{a}{b^2} y^2$

35. **(b) :** For a unit vector $\hat{n}, |\hat{n}| = 1$

$|0.5\hat{i} - 0.8\hat{j} + c\hat{k}|^2 = 1^2 \Rightarrow 0.25 + 0.64 + c^2 = 1$

or $c = \sqrt{0.11}$